Experimental Evaluation of Influence Factors for Single Geocell-Reinforced Sand

Sanat Kumar Pokharel, Graduate Research Assistant Civil, Environmental, and Architectural Engineering Department, the University of Kansas, Email: sanpok@ku.edu

Jie Han*, Ph.D., P.E. Associate Professor, Civil, Environmental, and Architectural Engineering Department, the University of Kansas. Email: jiehan@ku.edu

Dov Leshchinsky Professor, Department of Civil and Environmental Engineering, the University of Delaware, Newark, DE19719. Email: reslope@aol.com

Robert L. Parsons Associate Professor, Civil, Environmental, and Architectural Engineering Department, the University of Kansas, Email: rparsons@ku.edu

> Izhar Halahmi Chief Scientist, PRS Group Ltd., UK Email: izharhal@012.net.il

Submitted to the Geosynthetics Committee (AFS70)

Word Count: 6499 Number of words in text: 3999 Number of figures: 8 Number of Tables: 2

* the corresponding author

Presented at the US Transportation Research Board (TRB) Annual Meeting, Washington, D.C., January 11-15, 2009 **Abstract:** Geosynthetics have been in use for subgrade stabilization and base reinforcement for last 40 years. Over the years, research conducted on geosynthetic-reinforced base courses, especially with planar reinforcements, have shown marked benefits over unreinforced ones. Geocell, a three-dimensional geosynthetic material with interconnected cells, can be used to improve the properties of base courses by providing lateral confinement to increase strength and stiffness and reduce permanent surface deformation. However, the use of geocells for base reinforcement is limited due to lack of established design methods. Literature review has shown a significant gap between the applications and the theories for geocell reinforcement mechanisms outlining the need for more research. This research utilized simple loading equipment to evaluate the influence factors of single geocell-reinforced sand. The tests investigated the effect of influence factors (geocell shape and type) on the bearing capacity and stiffness of compacted sand. The experimental results showed that the geocell reinforcement increased the bearing capacity and stiffness and reduced settlement of the base course. The magnitude of improvement varied with the type of geocells.

Keywords: Geosynthetic reinforcement, geocell, bearing capacity, unpaved roads.

INTRODUCTION

An estimated 80% of all roads in the world are unpaved and majority of them are low-volume. According to AASTHO reports, approximately 20% of pavements fail due to insufficient structural strength. It is common practice for the low-volume road managers to use their limited resources for repair, maintenance, and rehabilitation (1). A sustainable option to overcome this problem is to develop an innovative pavement stabilization technique with a suitable reinforcement alternative that improves the overall structural strength, reduces operational costs, and minimizes maintenance requirements. Geosynthetic reinforcement is one of the established techniques of ground improvement for over 40 years (2)(3). Since the 1970s, geosynthetics have been used to improve the performance of both paved and unpaved roads. While planar reinforcements, such as geogrids and geotextiles, are commonly used for soil reinforcement at the subgrade-base interface or within the base course to increase the performance, the use of a three-dimensional interconnected honeycomb type of polymeric cells (commonly known as geocell) has been increasing. Use of geosynthetic materials as reinforcement has been reported to increase bearing capacity, reduce settlement, and minimize the amount of aggregate material required to extend the service life of roads.

Most of the research has focused on planar reinforcement and has resulted in several design methods (2)(3)(4)(5). More research is needed to develop such a design method for threedimensional interconnected geocells. The concept of lateral confinement by cellular structures dates back to late 1970s. The United States Army Corps of Engineers developed this idea for providing lateral confinement to improve the bearing capacity of poorly graded sand in 1970s (6). The predecessors of present geocells were known as sand grids made up of paper soaked in phenolic water resistant resin. Later metallic geocells, especially aluminum, were chosen on the ground of strength but they proved unfeasible because of handling difficulty and cost. The polymeric geocells currently in use eventually emerged as a sustainable alternative. High-density polyethylene (HDPE) is the most common polymer used to make geocells. Extruded HDPE strips are welded together to form the honeycomb. Geocells come in different shapes and sizes; there are variations in the type of material used, the aspect ratio, and the height and thickness of the cells. The geocell structures have been found to be effective in soil confinement and hence have a promising future for base reinforcement.

This paper discusses the results of plate load tests conducted to evaluate the influence factors for single geocell-reinforced sand. Laboratory tests for this research were carried out on different types of single geocell with poorly-graded Kansas River sand as the granular infill material. A set of laboratory tests were conducted to study the influence of the shape and the type of geocell material on the bearing capacity and stiffness of the geocell-reinforced sand.

PAST STUDIES ON GEOCELL REINFORCEMENT

As mentioned above, the pioneering studies on three-dimensional soil confinement cells in the 1970s were basically concerned with the feasibility of the confinement structure (6), later named as "Geocell". The research then considered base reinforcement on weak subgrade, which mostly revolved around the reinforcement mechanism, properties and geometry of the geocell, and infill material. The major concerns of those studies were the effects of geocell height to width (i.e., aspect ratio), tensile stiffness of geocell material, bearing capacity, conjunctive use with other planar geosynthetic reinforcement, loading type, base and subgrade materials, and density of

infill material. Yuu et al. (7) summarized past studies on geocell from triaxial compression tests, laboratory model tests, and field tests.

Rea and Mitchell (8) conducted both experimental and theoretical studies to investigate the reinforcement mechanisms and failure modes for geocell and suggested the optimum dimensions for cells relative to the size of the loading plate. Laboratory tests were also carried out by de Garidel and Morel (9), Jammnejad et al. (10), Kazerani and Jammnejad (11), Bathurst and Jarrett (12), Shimizu and Inui (13), Mhaiskar and Mandal (14), Bathurst and Karpurapu (15), Rajagopal et al. (16), Dash et al. (17), Latha and Murthy (18), Mengelt et al. (19), and Chang et al. (20) to investigate the benefits of geocell reinforcement and different influence factors. Bathurst and Karpurapu (15), Rajagopal et al. (16), and Latha and Murthy (18) confirmed an increase in the stiffness and strength imparted by the confinement effect of geocell reinforcement.

Jammnejad et al. (10) compared existing theoretical solutions for a soil-geocell composite and found that they are not satisfactory in predicting the performance and life of roads. Kazerani and Jammnejad (11) found geocell reinforcement to significantly improve the load-deformation and stress distributing characteristics of poorly-graded materials and outlined a design procedure for geocell reinforced structures based on allowable vertical compressive strain at the basesubgrade interface. The geocomposite mattress model showed great improvement in the bearing capacity and a stiffer geocell was found to double load bearing capacity at a certain rut depth for a given mattress thickness (12). Shimizu and Inui (13) found that the installation of a cell wall was efficient for increasing the bearing capacity of the ground with increased height and reduced cell area, and the extent of increase was correlated with the horizontal stiffness of the cell material. Dash et al. (17) demonstrated the advantages of the geocell as compared with other planar and randomly distributed mesh elements. This study (17) noted that the confinement by the geocell creates a better composite material, redistributes the footing load over a wider area, and reduces the settlement. The reinforced section showed marked improvement in bearing capacity and the increase was higher for higher cell height. Mhaiskar and Mandal (14) reported a better performance by geocell reinforcement compared to planar reinforcement for increasing the bearing capacity. Zhou and Wen (21) found that the geocell-reinforced sand cushion reduced the settlement by 44%.

A summary of the past studies on geocell reinforcement confirms that the geocell can provide confinement and increase the strength and modulus of infill material. There exist optimum values of the geocell height/width ratio and the loading area width/geocell width ratio, and the increase in bearing capacity depends on the quality of infill material. Han et al. (22) proposed a laboratory test method for evaluating planar geosynthetic-soil confinement based on the performance of the reinforced base, which is easy, quick, and inexpensive, applicable for all types of geosynthetics, and can simulate the interaction of geosynthetic with base course material, local deformation, repeated loading, and a wheel tracking motion. However, no design and test methods are available to incorporate all these factors for geocell-reinforced bases. Therefore, research is needed for developing reliable design and test methods for geocell-reinforced bases.

LOAD BEARING AND REINFORCEMENT MECHANISMS

As compared with the unreinforced base, the geocell-reinforced base can provide lateral and vertical confinement, tensioned membrane effect, and wider stress distribution. Due to the threedimensional structure, FIGURE 1 shows that the geocell can provide lateral confinement to soil particles within cells (23). The geocell provides the vertical confinement in two ways: (1) the friction between the infill material and the geocell wall and (2) the geocell-reinforced base acts as a mattress to restrain the soil from moving upward outside the loading area. The tensioned membrane effect is referred to as the tension developed in the curved geocell-reinforced mattress to resist the vertical load (16)(17)(21). Because the inclusion of the geocell and the confinement of the geocell would increase the stiffness of the reinforced base, the geocell-reinforced base with the higher stiffness has a wider stress distribution than the unreinforced base. The geocell-reinforced base exhibits bending resistance, tensile strength, and shear strength and intercepts the failure planes from the subgrade (21).

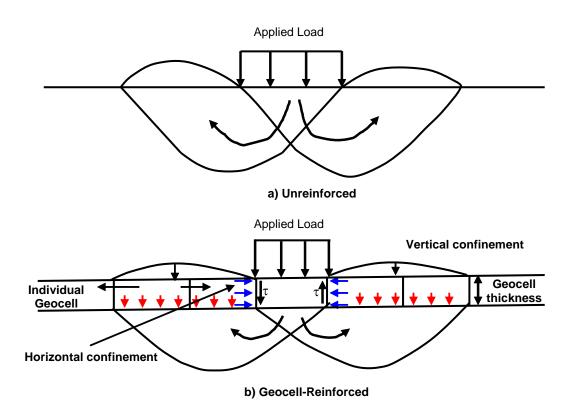


FIGURE 1 Unreinforced and geocell-reinforced soil behavior.

MATERIALS AND EQUIPMENT FOR THE EXPERIMENTAL TESTS

Geocell Types and Characteristics

Three types of geocell were used for the tests in this study. The properties of these geocells are given in TABLE 1 and their stress-strain curves are shown in

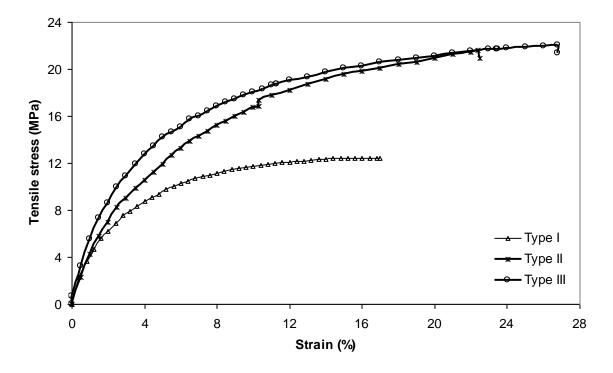


FIGURE 2. Type I geocell is made up of regular HDPE having tensile strength of 12.5 MPa. The other two types of geocell are made up of novel polymeric alloys having the same polymeric composition but different extrusion subroutines. The novel polymeric alloy is characterized by flexibility at low temperatures similar to HDPE with elastic behavior similar to engineering thermoplastic. Type II and Type III geocells have tensile strengths of 20.9 and 21.3 MPa, respectively. Field studies have shown that the measured strains in the field for geosynthetics are typically within 2%. The elastic moduli of three types of geocell at 2% strain are provided in TABLE 1. The stress-strain curves were measured at a strain rate of 10%/minute at 23° Celsius. The modulus of Type III is 1.5 times that of Type I. All three types of geocell used in this study did not have any perforation.

TABLE 1 H	Properties of	Geocells
-----------	---------------	----------

Type	Material	Thickness	Height	Tensile strength	Elastic modulus at
		(mm)	(mm)	(MPa)	2% strain (MPa)
Ι	HDPE	1.51	100	12.5	310
II	Novel polymeric alloy	1.1	100	20.9	350
III	Novel polymeric alloy	1.1	100	21.3	440

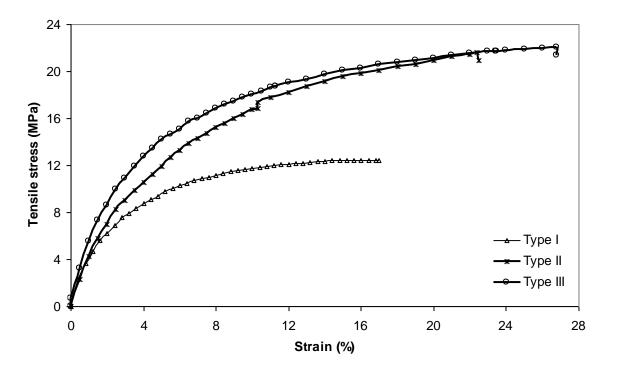


FIGURE 2 Tensile stress-strain curves of geocells.

Granular Base Material

Kansas River sand was used as the granular base for the tests. It is a poorly-graded sand having a minimum void ratio of 0.354, a maximum void ratio of 0.583, and a specific gravity of 2.65 at 20°C. The mean grain size (d_{50}) of this sand was 2.6 mm. The grain size distribution curve of Kansas River sand is shown in FIGURE 3.

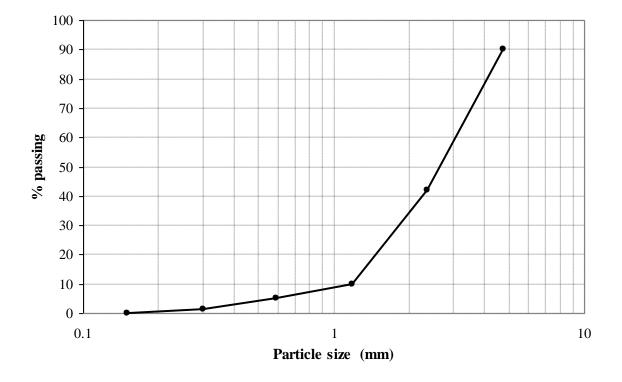


FIGURE 3 Grain size distribution curve of Kansas River sand.

Test Setup

Model tests were conducted in a medium-scale loading apparatus designed and fabricated at Department of Civil, Environmental, and Architectural Engineering at the University of Kansas. The loading system had a 15 cm diameter air cylinder with a maximum air pressure of 2100 kPa. The loading plate was 15 cm in diameter. FIGURE 4 shows the details of the test box, which was square and had a plan area of 3660 cm² with an adjustable depth. Geocell was placed at the center of the box and its shape and size depended on the designed layout of circular and elliptical shapes. All single cells in this study were 10cm high. For the test the geocell was filled and embedded in sand. The sand was placed into the box and compacted to 70% relative density in three layers, 5 cm each for the first two layers and the top layer of 2 cm. The compaction to 70% relative density was maintained in all tests. No subgrade existed for all the tests because the primary purpose of this research was to evaluate the influence factors for the single geocell-reinforced sand. Before these tests, a possible boundary effect due to the size of the test box was investigated. The tests showed that there is no boundary effect if the width of the square box is larger than 60cm, which was used in this study.

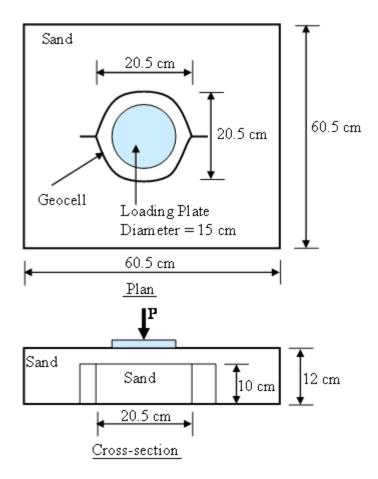


FIGURE 4 Test box for single cell tests with geocell at the center.

RESULTS AND DISCUSSION

Effect of Geocell Shape

Different shapes of geocell have been tested by others in the past. Most of them were either circular or box-shaped. Simizu and Inui (13) carried out studies on a six sided cell made of geotextile product. Rea and Mitchell (8) conducted tests on square-shaped paper grid cells to identify different modes of failure and arrive at optimum dimensions of the cell. In practice geocell are placed in a near circular pattern.

In this study, tests were carried out on elliptical and circular geocell shapes for geocell Types I, II, and III embedded in Kansas River sand. For the first set of tests, the geocell was laid out in an elliptical shape with the major diameter along the weld side equal to 26 cm and the minor diameter equal to 18.5 cm. At the higher load, cells with a shape that was initially elliptical changed to a near circular shape and with the major diameter along the weld side changing to 23.5 cm and the minor diameter to 20 cm. The shapes, before and after the tests, are shown in

FIGURE 5. The second set of tests used a circular layout for the cell with the diameter equal to 20.5 cm. For these tests, no appreciable change in the shape was observed during the test.

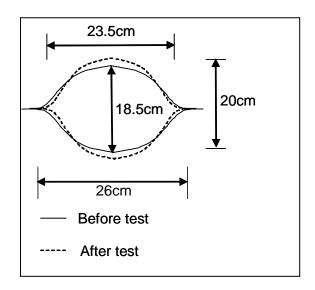


FIGURE 5 Change of geocell shape after test.

The results of the above tests are summarized in

FIGURE 6. In all cases the reinforced sections were found to perform better than the unreinforced section. The geocells starting with a circular shape showed stiffer and stronger responses than those starting with the elliptical shape in all three types of geocell-reinforced sand. FIGURE 6 also shows that the reinforced base had the stiffness and the strength on the order of geocell Type I, II, and III from low to high values. This order is consistent with that of the elastic moduli at 2% strain in Table 1.

Since there was no weak subgrade for any of the tests, the geocell was lifted up appropriately by 8 mm from the base after each test when the geocell was placed in an elliptical shape and 5 mm when placed in a circular shape. These data indicate that the infill material escaped from the bottom of the cell under the load. The improvement of the interface properties between the geocell and the infill can minimize the chance for the geocell being uplifted and is expected to further increase the bearing capacity of the reinforced base.

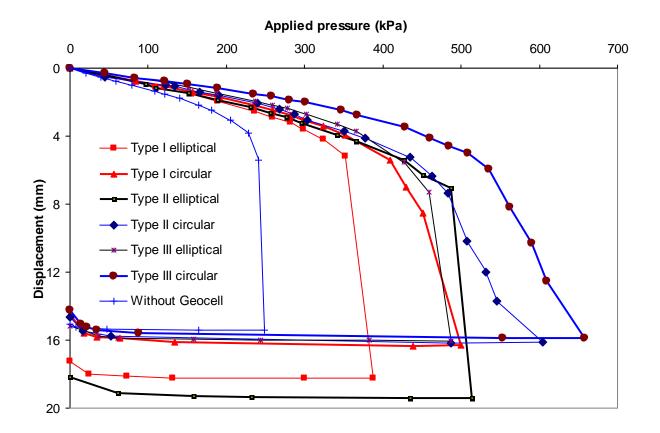


FIGURE 6 Pressure-displacement curves for unreinforced and reinforced bases by single geocell in an elliptical or circular shape.

Effect of Type of Geocell Material

The properties of geocell material (especially modulus) are reported to have an influence on the bearing capacity and stiffness of the geocell-reinforced base. For a given height and width to height ratio, the elastic modulus of the geocell plays a more important role than the seam strength (24). Mengelt et al. (19) reported a marked increase in the resilient modulus of cohesive soils (16.5 to 17.9%) caused by single geocell reinforcement but a minor increase in the case of granular soils (1.4 to 3.2%). The plastic deformation was found to decrease significantly for both cases.

To verify the influence of geocell properties on the bearing capacity and the stiffness of the geocell-reinforced sand, loading tests were carried out on all three types of geocell embedded in the sand. The unreinforced

base was taken as the baseline case for comparison. The increase in the bearing capacity and stiffness for two geocell shapes (elliptical and circular) for the case of geocell embedded in sand is clearly evident from the

ultimate loads and the initial slopes of the curves in

FIGURE 7 and

FIGURE 8. For all tests, the geocell-reinforced sand failed or yielded at approximately 5mm displacement, which is equivalent to 3.3% the diameter of the loading plate. The geocells initially with an elliptical shape failed abruptly while the geocells initially with a circular shape failed gradually. Due to the change of the geocell shape from the elliptical to circular one, the

sand particles inside the geocell had larger movement, uplifted the geocell, and resulted in a sudden failure. After each test, approximately 5 mm uplift for the initially circular geocell was measured, which was much less than approximately 8 mm uplift for the initially elliptical geocell.

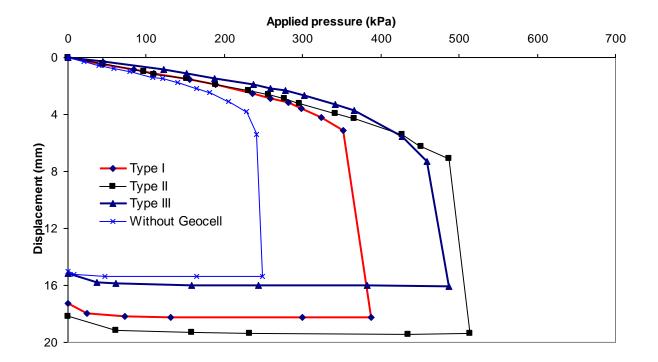


FIGURE 7 Pressure-displacement curves of unreinforced and reinforced bases by single elliptical geocell.

The above results can be expressed in terms of the improvement factor, where the improvement factor is defined as the ratio of the ultimate bearing capacity or stiffness of the reinforced base to the unreinforced base. As summarized in TABLE 2, geocell reinforcement increased the bearing capacity of sand by an improvement factor of 1.5 to 2.5 and the stiffness by a factor of 1.3 to 2.0 depending upon the type of geocell and the initial shape of geocell. These results are in good agreement with those obtained by Han et al. (25) earlier. The two types of geocell with novel polymeric alloy showed more improvement than the traditional HDPE geocell reinforcement. The performance of the geocell from the best to the worst is on the order of Types III, II, and I, which is consistent with the order of the elastic moduli. The geocell placed in a circular shape performed better in terms of ultimate bearing capacity and stiffness than that placed in an elliptical shape. Overall, geocell Type III placed in a circular shape was the most effective in increasing the ultimate bearing capacity and reducing the settlement.

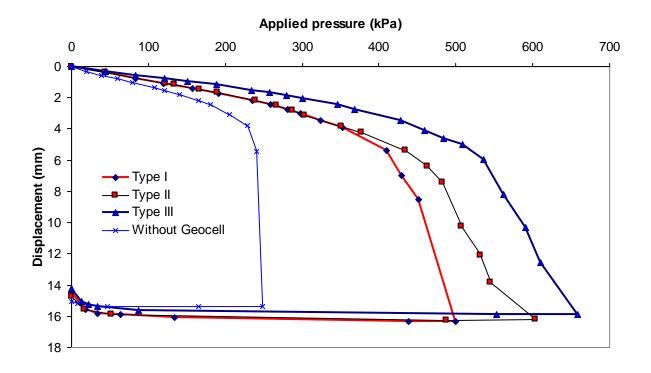


FIGURE 8 Pressure-displacement curves of unreinforced and reinforced bases by single circular geocell.

TABLE 2 Improvement Factors for Bearing Capacity and Stiffness of Geocell-reinforced
Sand over the Unreinforced Sand

	Improvement factor for shape of geocell					
Reinforcement type	Elliptical layout		Circular layout			
	Bearing capacity	Stiffness	Bearing capacity	Stiffness		
Type I	1.5	1.3	1.8	1.5		
Type II	1.9	1.3	2.0	1.7		
Type III	2.0	1.8	2.5	2.0		

Even though Type II and III geocells made from the polymeric alloy had 30% thinner walls, they performed better than the regular HDPE geocell. This result implies that the material specific properties are more important than the specific property multiplied by dimension (thickness in this case).

CONCLUSIONS

This paper presents the results of a laboratory study to investigate two key influence factors on the behavior of single geocells: the placement shape and the elastic modulus. The following conclusions can be made from this study:

- (1) The geocell placed in a circular shape had a higher bearing capacity and stiffness of the reinforced base than that placed in an elliptical shape.
- (2) The performance of geocell-reinforced bases depended on the elastic modulus of the geocell. The geocell with a higher elastic modulus had a higher bearing capacity and stiffness of the reinforced base. Type II and Type III geocells made from the polymeric alloy were found significantly better in ultimate bearing capacity, stiffness, and reinforcement potential relative to Type I geocell made from HDPE.
- (3) The improvement factor for the reinforced base over the unreinforced base ranged from 1.5 to 2.5 in terms of bearing capacity and 1.3 to 2.0 in terms of stiffness. The geocell with a higher elastic modulus had a higher improvement factor.

ACKNOWLEDGEMENTS

This research was funded jointly by the University of Kansas, Transportation Research Institute from Grant #DT0S59-06-G-00047, provided by the US Department of Transportation – Research and Innovative Technology Administration and PRS Group, Ltd, UK. This support is greatly appreciated. The loading apparatus used in this research was designed and fabricated by Mr. Howard Jim Weaver, the lab manager in the Department of Civil, Environmental, and Architectural Engineering (CEAE) at the University of Kansas (KU). Mr. Milad Jowkar, an undergraduate student, and Mr. Anil Bhandari, a graduate student, in the CEAE Department at KU provided assistance in the testing. The authors are thankful for their great help.

REFERENCES

- 1. Tingle, J.S., and S.R. Jersey. Empirical Design Methods for Geosynthetic-Reinforced Low-Volume Roads. *Journal of the Transportation Research Board*. No. 1989. Vol. 2, 2007, pp. 91-101.
- Giroud, J.P., and J. Han (a). Design Method for Geogrid-Reinforced Unpaved Roads. I. Development of Design Method. ASCE Journal of Geotechnical and Geoenvironmental Engineering, Vol. 130, No. 8, 2004, pp. 775-786.
- 3. Giroud, J.P., and J. Han (b). Design Method for Geogrid-Reinforced Unpaved Roads. II. Calibration of Applications. *ASCE Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 130, No. 8, 2004, pp. 787-797.
- Giroud, J.P., and L. Noiray. Geotextile- Reinforced Unpaved Road Design. ASCE Journal of the Geotechnical Engineering Division. Vol. 107, No GT9, 1981, pp. 1233-1254.
- Leng, J., and M.A. Gabr. Deformation-resistance Model for Geogrid-Reinforced Unpaved Road. *Journal of the Transportation Research Board*, No. 1975, Transportation Research Board of the National Academies, Washington, D.C., 2006, pp. 146–154.
- 6. Webster, S.L. Investigation of Beach Sand Trafficability Enhancement Using Sand-Grid Confinement and Membrane Reinforcement Concepts. *Report GL-79-20 (1). U.S. Army Engineer Waterways Experiment Station*, Vicksburg, MS, 1979.

- Yuu, J., J. Han, A. Rosen, R.L. Parsons, and D. Leshchinsky. Technical Review of Geocell-Reinforced Base Courses over Weak Subgrade. *GeoAmericas, Cancun, Mexico*, March 2 to 5, 2008.
- 8. Rea, M., and J.K. Mitchell. Sand Reinforcement Using Paper Grid Cells. *Regular Meeting- Rocky Mountain Coal Mining Institute*. 1978, pp. 644-663.
- 9. de Garidel, R., and G. Morel. New Soil Strengthening Techniques by Textile Elements for Low-Volume Roads. *Road and Railway Applications, Third International Conference on Geotextiles*, 1986, Vienna, Austria.
- 10. Jammnejad, G., G.H. Kazerani, R.C. Harvey, and J.D. Clarke. Polymer Grid Cell Reinforcement in Pavement Construction. *The 1986 International Conference on Bearing Capacity of Roads and Airfields*. September 16–18, 1986, Plymouth, England.
- 11. Kazerani, B., and G.H. Jammnejad. Polymer Grid Cell Reinforcement in Construction of Pavement Structures. *Section 1A, Unpaved and Paved Roads. Geosynthetic '87 Conference*, New Orleans, USA, 1987.
- 12. Bathurst, R.J., and P.M. Jarrett. Large-Scale Model Tests of Geocom Mattresses over Peat Subgrades. *Transportation Research Record 1188*, 1989, pp. 28-36.
- 13. Shimizu, M., and T. Inui. Increase in the Bearing Capacity of Ground with Geotextile Wall Frame. *Geotextiles, Geomembranes and Related Products*, Den Hoedt (ed.), Balkema, Rotterdam. 1990, pp. 254.
- 14. Mhaiskar, S.Y., and J.N. Mandal. Comparison of Geocell and Horizontal Inclusion for Paved Road Structure. *Earth Reinforcement Practice, Ochiai, Hayashi and Otani*. Balkema, Rotterdam, 1992.
- Bathurst, R.J., and R. Karpurapu. Large-Scale Triaxial Compression Testing of Geocell-Reinforced Granular Soils. *Geotechnical Testing Journal*, *GTJODJ*, Vol. 16, No. 32, 1993, pp. 296-303.
- Rajagopal, K., N.R., Krishnaswamy, and G.M. Latha. Behaviour of Sand Confined With Single and Multiple Geocells. *Geotextiles and Geomembranes*. Vol. 17, 1999, pp. 171 -184.
- 17. Dash, S.K., K. Rajagopal, and N.R. Krishnaswamy. Performance of Different Geosynthetic Reinforcement Materials in Sand Foundations. *Geosynthetics International*, 11, No. 1, 2004, PP.35-42.
- 18. Latha, G.M., and V.S. Murthy. Effects of Reinforcement Form on the Behavior of Geosynthetic Reinforced Sand. *Geotextiles and Geomembranes*, Vol. 25, 2007, pp 23-32.
- Mengelt, M.J., T.B. Edil, and C.H. Benson. Resilient Modulus and Plastic Deformation of Soil Confined in a Geocell. *Geosynthetic International*, Vol. 13, No 5, 2006, pp. 195-205.
- Chang, D T., C H. Chang, and S.W. Pai. Investigation of Bearing Capacity and Dynamic-Elastic Behavior of Mechanical Stabilization of Sandy Subgrade Using Geocells. *Transportation Research Board Annual Meeting CD-ROM*, 2007.

- 21. Zhou, H., and X. Wen. Model Studies on Geogrid or Geocell-Reinforced Sand Cushion on Soft Soil. *Geotextiles and Geomembranes*. 2007, Vol. 26, Issue 3, 2008, pp. 231-238.
- 22. Han, J., Y. Zhang, and R.L. Parsons. Development of a Performance-Based Laboratory Test Method for Evaluating Geosynthetic-Soil Confinement. *Transportation Research Board* 87th Annual Meeting January 13–17, 2008, Washington, D.C.
- 23. Hufenus, R., R. Rueegger, R. Banjac, P. Mayor, S.M. Springman, and R. Bronnimann. Full-Scale Field Tests on Geosynthetic Reinforced Unpaved Roads on Soft Subgrade. *Geotextiles and Geomembranes*, 24, 2006, pp. 21-37.
- 24. Mhaiskar, S.Y., and J.N. Mandal. Investigation on Soft Clay Subgrade Strengthening using Geocells. *Construction and Building Materials*, Vol. 10, No. 4, 1996, pp. 281-286.
- 25. Han, J., X.M. Yang, D. Leshchinsky, and R.L. Parsons. Behavior of geocell-reinforced sand under a vertical load. *Journal of Transportation Research Board*, in press. 2008.